

Cognition Technology, Accuracy Significance and Contracts' Incompleteness

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Abstract

This paper studies the effect of accuracy of cognition technology on the completeness of the contracts. Principal invests to find the future state as well as the blueprint of the design. The blueprint, however, may not be accurate and comprehensive. We use two classes of contracts; *hiring* in which principal and agent work together just for developing the design and *joint production* in which they develop and produce the product jointly, to explain accuracy significance on the cognition investment as well as incompleteness of contract. In particular, we find that accuracy of the blueprint is one of the important factors of the cognition investment and incompleteness. We also find that accuracy is the only driving force of the relative incompleteness of each contracts. And in line with empirical works, we explain why we see different levels of completeness in joint production contracts by using this notion.

Keywords: cognition investment, accuracy, contract incompleteness

JEL classification: D23, D82, D86, L22.

1 Introduction

In the standard contract theory the states of nature are known with a probability measure on every specific state and specifying a contract is the burden of finding the allocations

*I am grateful to John Nachbar for helping me from the time this paper was just a vague idea.

which satisfy some incentive conditions. There is no cost for economic agents to see future states. However, a deviation from this standard were some works which include the cost to know what is the future state exactly. This paper is in the same environment with the difference that the cognition investment is not perfect in a sense that even though the parties find future state correctly but their specifications are not comprehensive. There is a probability that they can not develop the appropriate design based on their finding from cognition investment. And cognition investment not only determines the probability of finding future state but also it affects the accuracy levels of the outline to be developed.

The main concern in this paper is that what is the contract form offered by the principal to the agent and what are the equilibrium behaviors of players in this situation? In particular we focus on the two different scenarios and their related contracts. In the first scenario, the principal invests to find true future state and offers contract to agent for just developing stage of the design. In the second scenario principal offers a contract, after cognition investment, which they work together both in the developing and producing stages. We call these cases *hiring* and *joint production*, respectively. We characterize the conditions under which each contract would be offered. We use same notion of completeness of contract as Tirole(2009), which states that the higher level of cognition investment, the more is completeness of the contract, and compare the completeness of each of the contracts. One of the basic objective of this study is to understand how imperfect cognition technology affects on the contract form and cognition investment.

We introduce the notion of accuracy to go one step further in the literature and see what are the consequences of imperfection in the cognition technology on the equilibrium behaviors and contract design. The imperfection goes back to the idea that even if one of the agents invest to verify what is the future state the imperfection of cognition technology results in some specifications of the future states which are not perfect. The accuracy level captures this notion. That is the higher the accuracy level more complete is specifications of future states.

We consider the situation where just principal invests to find what is the true future state. Initially, both principal and agent are aware of specific design which can be thought of existing product in the market. The principal is sure that designing based on the existing product outline and using it for final good may not be appropriate for future state. However, he does not know the future state until he invests to find future state. So principal needs to invest about finding what is the true future state. Along with searching to find future true state, he will prepare an outline for the developing of design.

The act of searching for future state is not only finding future state, but also it affects on the accuracy of the outline which design will be developed based on that. That is high level of cognition investment by principal would result in higher level of accuracy. After cognition investment, principal offers the contract (which could be either hiring or joint production). If the outline is the existing design they develop intended design but if the outline is the non-existing one they may end up to some development which is not appropriate for the next stage. Since the outline for non-existing design is not comprehensive, the developed design may not satisfy all requirements in the final stage.

We interpret the complexity of the production as probability of developing appropriate design. So, as explained above this interpretation of complexity refers to the probability of developing non-existing design. The higher probability of getting appropriate design, the lower is the complexity of development and therefore production. In the basic model this complexity is exogenous but in the generalized model the cognition investment results in two different levels of accuracies with known probability distribution. These accuracy levels affect the probability of developing appropriate design. The relation is such that the higher the level of cognition investment, the lower is the difference between accuracy levels and of course the higher are the both accuracies levels. This setting not only makes a clear relation between precontract investment and after contract effect of it but also it captures *the complexity of the product design*.

As a clear example for the above explanation you can imagine Company X (as principal) who invests to find what is the true future state for computer monitor (suppose the existing one is CRT monitor and the new one is the LCD monitor). After cognition investment he will end up with type of monitor needed in future state and outline which development is based on that. Principal can develop the monitor by hiring the agent just for the developing stage and deals with the final stage by himself or they can develop and produce the monitor together. Our model will explain which contract form would be chosen and what is the effect of accuracy on the equilibrium behaviors and cognition investment.

In the following sections; we first explain the related literature in section 2. In section 3, we explain the basic model and in section 4 we analyze the generalized model. Conclusion and appendix come in sections 5 and 6, respectively.

2 Related Literature

This paper is more related to the Tirole(2009) and Bajari and Tadelis (2001). In the following I will explain these two papers in more details.

Tirole (2009) is one of the closest papers to this one. He analyzes the implication of cognition cost on the incompleteness of contract in a buyer-seller setting. One of the key forces of investment by parties is not only the avoidance of ex post contract adjustments but also rent seeking. He concludes that because of rent seeking behavior contracts basically may be too complete. He finds that both relational contract and integration have negative effects on the cognition investment (i.e. contract completeness). Our model is based on Tirole (2009) with some modifications. Different from Tirole(2009), we explain the decision of producing the product individually or jointly. Here the complexity in design stage is the other component which plays role on top of the the other forces in Tirole(2009). So we will explain the situation in which the principal decides not only on the cognition levels but also on the type of contract. Here just principal invests before offering the contract. The other difference from all other related papers (including Tirole(2009)), is that the cognition investment affects the accuracies levels. This will push us to solve the whole game backward.

Bajari and Tedelis (2001)(BT henceforth) pioneered in finding incompleteness of the contract endogenously in a buyer-seller setup. Based on the two different types of contracts in construction industry, they tried to answer that under which conditions any type of contracts is chosen? In their model tension between providing ex ante incentives and avoiding ex post transaction costs due to costly renegotiation is driving force to find the right contract. Their main finding is that more complex products (i.e. more states of nature) have less complete design and are more likely to be procured using incentive contracts. They find that higher level of integration is aligned with lower level of design investment and higher complexity of the product will result in more integration decision. This paper differs from BT as follows. In BT The assumption is that for any product, its complexity is known and finding the complexity is just matter of paying the cost of specific design. In their paper the implicit assumption is that the parties can develop appropriate design if they find the outline of design. However, in reality, the complexity is twofold. First, the parties may not be completely informed of the future states even after investing to find them. And second, they may not even be able to deliver appropriate design because the outline is not comprehensive. While BT focus on the first type of complexity, the focus in this paper is on the second type. Moreover, this paper analyzes

different contractual setup in which principal not only incurs cognition investment but also he is involved in the producing of product.

In Lewis and Sappington (1997), they consider a standard procurement model and analyze the incentive contract under which the agent can invest to acquire valuable information. They show this condition changes standard contract and cause some distortions which can be avoided if principal assigns the planning and executing the jobs to two different agent. The difference here is that just principal gathers information before the contract is offered and the contractual design is different. And agent's effort is contractible. In Cremer, Khalid,Rochet (1998 a,b), they analyze the situation in which agent can invest to find some information about future state (about cost function) *before* and *after* of receiving the contract offer. The difference here is that principal will invest to find true future state. Agent does not know about the cognition investment level. The other difference between our model and theirs is that cognition investment by principal has effect on the accuracy level of outline which is provided by the principal.

3 Basic Model

The game has three stages, in **stage-0**, principal(P) searches for appropriate design. The structure of finding the design is like Tirole(2009). There are two states which just one of the designs is appropriate upon their realization. Suppose with probability of $1 - \rho$ the state is B and with ρ it is B' . If P invests to find what the true future state is; he will find it with probability b provided the true state is B' and find nothing with probability $1 - b$. He finds nothing about what is the future state if true state is B . So, at the end of stage-0 principal either has outline of the design for state B or state B' . Clearly, if he finds that true future state is B' he has outline of design B' and he has outline of design B otherwise. The cost of investment is $T(b)$ which we assume is increasing and convex, $T'(0) = 0, T'(1) = \infty^1$.

In **stage-1**, principal and agent(A) get together and develop the design based on the outline from stage-0. P incurs cost $c_1 > 0$ and A can choose the effort level e in which $e \in [0, 1]$ with cost $c(e), (c'(e) > 0)$. The result of their collaboration however, may **not** be appropriate at the final stage. There is a probability $1 - r^2$ that developing based on

¹An extensive form game of this model can be found in the Appendix-C

²In section 4, we endogenize the notion of accuracy. Higher levels of r results from higher level of accuracy and vise versa. We use r to capture the complexity of the production whether the underlying accuracy level is exogenous or endogenous.

the outline B' turns out to be inappropriate at the beginning of stage-2. This probability shows the uncertainty which is contained in the development stage. It can be thought of since the outline is not comprehensive; developing based on that loses some features of the appropriate design. It is natural that both parties will not know the appropriateness of the design before final stage has realized. This probability distribution is common knowledge between both players. If the design's outline is B , there is no uncertainty about developing design and they will develop a design which is appropriate for state B . So what is happening here is that even if P gets the outline of design by his cognition investment but there is probability that he will end up with inappropriate developed design in his hand. This uncertainty involved in developing stage is exogenous and can not be controlled by any parties. While cognition investment, b , is chosen endogenously. The timing is as below;

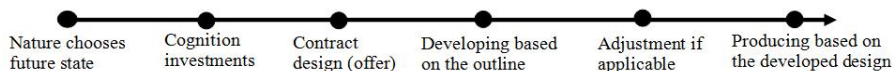


Figure 1: Timeline of events.

In **stage-2**, the developed design will be used to produce final good. In our real example, in this stage, the developed software will be installed on the hardware. The story in this stage is as follows; if the software is appropriate the value of product is V but if the software is not appropriate it will result in value $V - \Delta$. If P produces the product by himself he will not be able to improve the software. But if P produces the product jointly with A (same agent in previous stage) there is possibility for the software to be improved (adjusted) to capture the loss of value Δ by paying the adjustment cost.

So in stage-2, P can produce the product by himself or jointly with A. If P produces the good by himself (call this scenario **Hiring**), that is P (as a company) can use the developed design {of the joint work with A (the other company)}, produce the product, sell the product, and get the value $V - c_P$ if the design is appropriate. That is if they develop design B' and the realized state (at the beginning of stage-2) is B' , P gets $V - c_P$ and if they developed based on outline B and state is B he also gets $V - c_P$. However, if the developed design is inappropriate for state he will lose money. For example, if they develop based on design B and the realized state is B' he just gets $V - c_P - \Delta$ where $\Delta > 0$. Here c_P is cost of P to produce the good from two stages, i.e. $c_P = c_P^1 + c_P^2$. So if P produces alone he gets the whole value of the product in both combinations of developed design and realized state (appropriate or inappropriate).

However, P could produce the good jointly with A (call this scenario **Joint Production**). In this scenario, they use their developed design from stage-1 and produce the good using that. In this case they can improve the product if developed design is inappropriate. To do that, they should negotiate about the improvement. For the time being, let assume that in the joint production, the negotiation procedure is similar to Tirole (2009). That is P gets σ portion and A gets $\beta = (1 - \sigma)$ portion of good's net surplus. So, their payoffs are as follows; if they produce based on design $B(B')$ and the realized state (at stage-2) is $B(B')$ then P gets $\sigma(V - c_P - c_A)$ and A gets $\beta(V - c_P - c_A)$. If the outline of design is B' and the realized state is B' but they developed inappropriate design they have the opportunity to improve the product (before distributing in the market) together. In this situation P gets $\sigma(V - c_P - c_A - a_1)$ and A gets $\beta(V - c_P - c_A - a_1)$. And also if the developed design is B and the realized state is B' they have the opportunity to improve the product. In this situation P gets $\sigma(V - c_P - c_A - a_2)$ and A gets $\beta(V - c_P - c_A - a_2)$. Here c_P is P's total cost of production and $c_A = c_A^1 + c_A^2$ is the A's total cost of production. a_1 and a_2 are *adjustment costs* in which $a_2 > a_1$ and $\Delta > a_2$. The relation between a_2 and a_1 shows that if they develop the design based on *correct* outline then it costs less to improve inappropriate design. $\Delta > a_2$ means that the recovering value of adjustment is more than the adjustment cost. One of the differences (among the others) between these two cases stems from the fact (in this model maybe assumption) that if the design of the product is not appropriate then P can not improve it by himself. That is he can **not** improve it in hiring contract and distribute the good in the market. However, in the joint production contract they can fix it with the total cost of a .

Let look at the trade-off which P encounters to decide about interaction with A. In the stage-2; if P produces the product alone he will get the value of $V - c_P$ for right developed design-state match. But he can not fix the losses Δ in developed design-state mismatch. Now the question is that is it worth to produce jointly with A to prevent losses (i.e. Δ)? To make this decision P takes into account not only the gains from getting capability (ability of jointly adjusting) to adjust the inappropriate developed design to appropriate one but also the losses from the portion of the value of the good which goes to the A. We will do this analysis in section 3.3. In the following subsections we start with analyzing hiring and joint production, respectively.

3.1 Hiring

Let look at the situations where both players end up in this game;

1. P invests b and finds B' outline and the developed design is appropriate; (ρbr) .
2. P invests b and finds B' outline and the developed design is inappropriate; $(\rho b(1-r))$.
3. P invests b and does not find anything when realized state is B ; $(1-\rho)$.
4. P invests b and does not find anything when realized state is B' ; $(\rho(1-b))$.

The terms inside of parentheses refer to the probability of realization of each situation.

Since we do not have any agency problem (no private information) in A's behavior, P offers the wage which makes A indifferent. That is P gets all surplus of production. So P offers wage contract $w = c_A^1$. To choose the optimal level of cognition investment, P solves the following problem;

$$\max_{\{b\}} \{-T(b) + \rho br(V - c_P) + \rho b(1-r)(V - \Delta - c_P) + \rho(1-b)(V - c_P - \Delta) + (1-\rho)(V - c_P) - c_A^1\}$$

The following equation gives the optimal level of b_1^* ;

$$-T'(b_1^*) + \rho r(V - c_P) + \rho(1-r)(V - \Delta - c_P) - \rho(V - c_P - \Delta) = 0$$

Solving this equation results in $T'(b_1^*) = \rho \Delta r$. By the characteristics of $T(\cdot)$, we can safely say that higher level of uncertainty in development stage (lower r) will result in lower level of cognition investment. However, higher loss value of inappropriate design (higher Δ) positively affects the cognition investment.

3.2 Joint Production

In this scenario, P not only develops the design but also produces the product in collaboration with A. Since the adjustment of the good is not doable without collaboration of two parties, they negotiate in stage-2. The negotiation procedure is as explained above. We assume that at the end of stage-0, P offers the contract (similar to hiring). In a similar paradigm of Tirole (2009) the decision on the contract (which here is going to be the wage) is as follows;

First, suppose P finds B' at the end of stage-0. Let $c = c_P + c_A$, P would get

$$r[\sigma(V - c)] + (1-r)[\sigma(V - c - a_1)]$$

by offering $w(B')$ he receives

$$r[V - c_P - w(B')] + (1-r)[V - \Delta - c_P - w(B') + \sigma(\Delta - a_1)]$$

equating these two terms gives us the $w(B') = \beta(V - c - (1 - r)\Delta) + c_A$. The point here is that because of uncertainty involved in the development of design, parties will have to negotiate to improve the design with probability $(1 - r)$. So in the course of finding $w(B')$ they take into account this possibility. Note that if P has the all bargaining power in negotiation stage ($\beta = 0$) then $w(B') = c_A$. i.e. A just gets her total cost (no sharing in the production surplus). Now suppose P does not find anything after he finished his search for finding future state. Therefore, they will develop based on the outline of design B . Let define

$$\hat{\rho}(b) = \frac{\rho(1 - b)}{1 - \rho b}$$

as the posterior probability which the B developed design is inappropriate. The intuition for this posterior probability is that the higher cognition investment, the lower is the probability that P does not find future state and therefore the lower is the $\hat{\rho}$. Similar to above, let $w(B)$ be the wage offer when P finds nothing after his search for true state. Now let look at what P would get;

$$(1 - \hat{\rho})[\sigma(V - c)] + \hat{\rho}[\sigma(V - c - a_2)]$$

and what he gets after offering this offer;

$$(1 - \hat{\rho})[V - c_P - w(B)] + \hat{\rho}[V - \Delta - c_P - w(B) + \sigma(\Delta - a_2)]$$

equating these two terms gives us the offered $w(B) = \beta(V - c - \hat{\rho}\Delta) + c_A$. Note that this wage structure is similar to when P finds that future state is B' . Both of them are taking into account the possibility of negotiation for the inappropriate developed design. This wage contract is a payoff equivalent contract with the sharing rule.

Before the cognition investment, P will choose it optimally by solving the following problem;

$$\begin{aligned} & \max_{\{b\}} \{-T(b) + \rho b r(V - c_P - w(B')) + \rho b(1 - r)(V - \Delta - c_P - w(B')) + \\ & \sigma(\Delta - a_1)) + \rho(1 - b)(V - \Delta - c_P - w(B) + \sigma(\Delta - a_2)) + (1 - \rho)(V - c_P - w(B))\} \end{aligned}$$

Substituting $w(B)$ and $w(B')$ and taking derivative with respect to b we get³

$$T'(b_2^*) = \rho\sigma(a_2 - (1 - r)a_1) + \rho\beta\Delta(1 - \hat{\rho})$$

Note that the level of b_2^* has positive relation with r , that is the higher level of complexity of developing stage is aligned with lower cognition investment. Now let define

³The algebra have been moved to Appendix-A.

the completeness of the contract based on the value of b . Higher level of b means higher probability of correct outline and lower ex-post negotiation for adjusting design. We call a contract is *more incomplete* if the probability of developing design B is higher. Having this we can infer about the completeness of the two type of contracts (scenarios). The following proposition characterizes the conditions under which each of the contract is more incomplete.

Proposition 1 *For high level complexity $r < \beta(1 - \rho)$ the hiring of agent is always more complete than joint production and for low level of complexity $r > \beta$ hiring the agent is more complete if only if $\Delta > \frac{\sigma(a_2 - (1-r)a_1)}{r - \beta}$.*⁴

Proof Since $T(b)$ is strictly increasing and convex, if $T'(b_2) > T'(b_1)$ then $b_2 > b_1$. From the values of both $T'(b_1)$ and $T'(b_2)$ we have;

$$\rho\sigma(a_2 - (1 - r)a_1) + \rho\beta\Delta(1 - \hat{\rho}) > \rho\Delta r$$

This gives us $\Delta(r - \beta(1 - \hat{\rho})) < \sigma(a_2 - (1 - r)a_1)$. We know $\hat{\rho}(0) = \rho$, $\hat{\rho}(1) = 0$, and $a_2 > a_1$. So the right hand side of below equation

$$\Delta < \frac{\sigma(a_2 - (1 - r)a_1)}{r - \beta(1 - \hat{\rho})}$$

is always negative when $r < \beta(1 - \rho)$ and positive when $r > \beta$. Hence in the former case $b_1^* > b_2^*$ and in the later one $b_1^* > b_2^*$ only if $\Delta > \frac{\sigma(a_2 - (1-r)a_1)}{r - \beta}$. ■

We can interpret the levels of r as the levels of complexity in the project. That is the higher value of r means that the probability of delivering appropriate design is higher which we can interpret it has lower level of complexity to develop. Proposition 1 says that if the level of complexity is high (low r) then P's cognition investment is less in case of offering joint production contract instead of just hiring agent. This is independent of how much is the risk of the project (value of Δ) and also speciality gains ($a_2 - (1 - r)a_1$). However, he takes into account both of these values if the complexity level of developing the design is not high. The lower (higher) the risk(speciality gain), the more complete is the joint production. So this simple analysis tells us that completeness of the contract in joint production respect to hiring the agent depends on the complexity of the developing stage and also on the speciality gain. This may be in contrast to what comes in mind in the first glance that the value of the risk determines completeness of each contract. It also indicates that joint production is not necessarily more incomplete.

⁴Please see the appendix-A for the cases when $\beta(1 - \rho) \leq r \leq \beta$.

3.3 Contractual Design

One of the differences with previous studies is that we consider different types of contracts here. For example consider the other scenario in which principal offers long term contract. The interesting observation here is that in all types of contracts two different forces work against each other. Principal should decide based on the trade off between gain (loss) of hiring the agent and offering him a joint production contract. Let define contract in hiring of agent as $\zeta_1 : e \times \Theta \rightarrow \mathfrak{R}$ and joint production as $\zeta_2 : \Theta \rightarrow \mathfrak{R}_P \times \mathfrak{R}_A$ where $\Theta = \{B, B'\}$ is state space. Define Net map $\aleph : \zeta_1 \times \zeta_2 \times \theta \rightarrow \mathfrak{R}$ which gives the net gain (loss) from offering c_2 instead of c_1 . So now our job is to characterize this map to find under which conditions each contract would be chosen.

In the first step let look at \aleph more closely and observe that \aleph depends on;

- Δ ; Loss of mismatch between developed design and state
- $a_{1,2}$; Adjustment costs
- $b_{1,2}$; Cognition investment levels
- ζ_1 ; Contract of hiring
- ζ_2 ; Contract of joint production
- $\{B, B'\}$; States of nature

This map takes two different types of contracts and compares gains from developed design-state match and losses from developed design-state mismatch in these two contracts. So it essentially compares the gains and losses from one contract to the gains and losses of other one. So if it is positive, it means that joint production is better than hiring and vice versa.

The other observation is that if principal offers the contracts, he would offer the optimal contract in each cases. So we should focus our analysis in domain of the contracts (in both cases) that are optimal. In previous sections we discussed the optimal contracts in both cases so now we analyze under which condition P chooses each contracts.

Now based on previous sections we can define net function as

$$\aleph(c_1, c_2, B, B', a_1, a_2, \Delta) = \{-T(b_2^*) + (V - c) - \rho\sigma\Delta\gamma - \beta(V - c) + \rho\Delta\gamma + \rho\sigma\lambda + \rho\Delta r b_2^*\} \\ - \{-T(b_1^*) + (V - c) + \rho\Delta r b_1^*\}$$

Where $\gamma = (b_2^*(1-r) + (1-b_2^*)\hat{\rho})$ as the probability which the design is not appropriate and needs to be negotiated in joint production and $\lambda = (b_2^*(1-r)(\Delta - a_1) + (1-b_2^*)(\Delta - a_2))$ as expected surplus in case of negotiation on inappropriate design (The algebra are in the Appendix-A). Now depending on the values of \aleph , P will choose hiring or joint production. In case of $\aleph = 0$ he is indifferent between two cases.

Proposition 2 *Principal will choose joint production if $\aleph > 0$, and hiring if $\aleph < 0$ and he is indifferent between two cases if $\aleph = 0$.*

As we explained in the introduction there is trade off between choosing any of these two cases. In proposition 1 we characterized the conditions of relation between two cognition levels. So, principal will choose joint production if

$$\{\rho\sigma\lambda + \rho\Delta r(b_2 - b_1) + \beta\rho\Delta\gamma\} + \{(-T(b_2) + T(b_1)) - \beta(V - c)\} > 0$$

The gains from choosing joint production are as follows; P's portion of expected surplus in case of inappropriate design $[\rho\sigma\lambda]$, gains from working on right design $[\rho\Delta r(b_2 - b_1)]$, and agent's portion of expected enhance value of the product $[\beta\rho\Delta\gamma]$. The costs of this trade off are as follows; cost from higher cognition investment $[-T(b_2) + T(b_1)]$, and loss of some part of surplus which goes to agent $[\beta(V - c)]$.

Let look at two extreme case in which in one of them $b = 0$ and in the other $b = 1$. When $b = 0$, it means that nothing is invested for finding future state. In this case principal chooses joint production if $\rho\sigma(\Delta - a_2) > \beta(V - c - \rho^2\Delta)$. And similarly in the case of perfect information about future state when $b = 1$, he chooses joint production if $\rho(1 - r)(\Delta - \sigma a_1) > \beta(V - c)$. That is principal will look at whether the gain from joint production in case the design needs to be adjusted is higher than the loss in case the design does not need to be adjusted. The former result is similar to the BT(2001), while the latter one is different from their result. Their result indicates that hiring is always chosen when $b = 1$ while in our model this is not the case. The difference stems from the possibility of imperfection cognition investment in which even if you get to know perfectly what is the future state but you develop a design which is not appropriate for future state.

One thing which is far from grasping by initial trade off analysis is the inclusion of $\beta\rho\Delta\gamma$ in favor of choosing joint production. That is, in the process of choosing the scenarios, principal takes into account the externality of offering this choice to agent as well. In the next section we will explain the effects of different components on contractual design.

3.4 Discussion

In this section we look at comparative statics. First, from both cases we know that the higher probability of developing appropriate design (higher r), the higher is the optimal value of cognition levels, b . In particular we have $\frac{\partial T'(b_1)}{\partial r} = \rho\Delta$ and $\frac{\partial T'(b_2)}{\partial r} = \rho\sigma a_1$. This is quite intuitive since higher value of r (lower developing complexity) gives more incentive to P to invest more and find design B' . Second, same argument works for the loss value in case of inappropriate developed design (Δ). The higher the loss value resulted from inappropriate developed design the higher is the cognition investment. Principal is willing to invest more to know what is the true future state if the loss of developing inappropriate design is higher.

The effect of these parameters on the decision of choosing the contract is as follows; first we observe that the higher the probability of developing appropriate design (higher r) the lower is the possibility which P chooses joint production. In particular we have $\frac{\partial \mathfrak{N}(\cdot)}{\partial r} = \rho(\sigma b_2^* a_1 - \Delta b_1^*)$. This shows that when the developing stage is highly complex (then $b_2^* < b_1^*$ by prop.1), marginal decrease in the complexity increases tendency to choose hiring contract. The intuition is that higher level of cognition investment in hiring contract when the developing stage is very complex will result in higher level of gaining. The effect of loss when the developed design is not appropriate Δ , on the choice is not straight forward. In particular we have

$$\frac{\partial \mathfrak{N}(\cdot)}{\partial \Delta} = \beta\rho\gamma + \rho r(b_2^* - b_1^*) - \rho\sigma b_2^*(1 - r)$$

this shows that Δ can have both positive and negative effects on choosing the contract. This preliminary result is actually in line with some empirical studies. While previous theoretical studies show that the higher the risk (higher Δ), the higher is the *joint production* but this model tells us that under some condition it may be the opposite. Empirical studies show that there are both positive and negative effects with respect to risk(Lafontaine and Slade (2007))⁵.

4 Generalized Model

Empirical evidences (Ryall and Sampson, 2003) show that the completeness of the contract differs even in special type of contract. What can explain different level of completeness in contracts? To explain this question in a theoretical framework we proceed this section by

⁵These studies are in different industries.

asking what would be the equilibrium behavior of players if cognition investment affects after contract behaviors? In other words, suppose there are different principals in a specific industry. If different principals with different level of cognition investments end up with outline B' then the natural question is that do the different level of cognition investments affect the *quality* of their outlines? The implicit assumption in the literature is that there is no difference among them.

However, if the idea of cognition investment is to find what is the true future state then it is reasonable that higher investment on cognition rises to *better* description of future states. So what is the optimal behavior of principal if the level of cognition investment affects the description of the future state, namely **accuracy** of the outline? Having these questions in mind, we continue by the following modifications of the basic model to capture this idea.

Now, suppose that the amount of investment of P not only determines the probability of finding the true future states but also it affects on the accuracy of the outline which he provides. So when P gives outline to A there is an agreement on how precise is the outline. The accuracy of the outline is *explained* by P in which higher value of b indicates higher level of accuracy. After that they start to work with each other, since principal's cognition investment level is not verifiable and also hidden information, agent does not know the value of b . Hence agent will choose her effort level based on the reported accuracy level. She will choose the effort level optimally which will be explained soon. It is natural to think that the more accurate outline will lead to appropriate developed design with higher probability. And of course the probability of appropriate design will be higher if agent shows higher effort.

Let look at what are the differences with the basic model. If after searching for the future state, P does not find any thing they will work for developing design B . However, if P finds future state is B' and gives outline of B' they will work on developing the design. The probability of getting appropriate design, r , is $r = eb_i\hat{r}$ which indicates that both cognition investment of principal and effort choice of agent are necessary to develop an appropriate design. Here e is agent's effort level such that $e \in [0, 1]$, $b_i \in \{\underline{b}, \bar{b}\}$ is the accuracy level and $\hat{r} \sim U[0, 1]$.

For any level of cognition investment b , there are two levels of accuracies. $\bar{b}(b)$ which is the outline with high accuracy and $\underline{b}(b)$ with low accuracy. As it seems natural the difference between these two levels of accuracy diminishes and at $b = 1$ we have $b = \underline{b} = \bar{b} = 1$. This part of the model can be illustrated by below figure.

In the model agent can not see the cognition investment level, b , of principal. However,

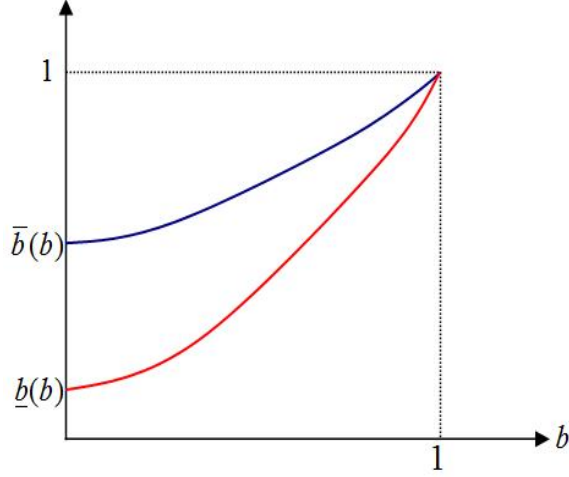


Figure 2: The accuracy levels as a function of b .

she has information about the accuracy levels of outlines in the market. To be clear, agent knows that there are two levels of accuracies with probability of α is \bar{b} type and $1 - \alpha$ is \underline{b} type. However, she does not know which type is the principal who deals with her. In the following sections we will explain first the equilibrium (or equilibria) in both scenarios and next we will explain contract design.

4.1 Hiring

In this case principal will offer a wage contract in which he pays the wage to the agent. The wage in the case of high type or low type principal will depend on the effort level. Since principal pays the agent according to the cost of effort levels then he will offer a wage contract such that it maximizes the expected payoffs. So principal will *set* wage levels such that

$$\begin{aligned} \max_{\{\underline{e}, \bar{e}\}} \{ & -T(b) + \rho b r (V - c_P) + \rho b (1 - r) (V - \Delta - c_P) + \rho (1 - b) (V - c_P - \Delta) + (1 - \rho) (V - c_P) \\ & - \rho b [\alpha c_A^1(\bar{e}) + (1 - \alpha) c_A^1(\underline{e})] - (1 - \rho b) c \}^6 \end{aligned}$$

Here $r = \frac{1}{2}[\alpha \bar{e} \bar{b} + (1 - \alpha) \underline{e} \underline{b}]$ and optimizing over effort levels gives the wage levels. The wage amount for any level of cognition investment is given by $\underline{w} = c_A^1(\underline{e})$ and $\bar{w} = c_A^1(\bar{e})$. The optimal effort level is derived by principal as $c_A^1(\underline{e}) = \frac{1}{2}(\underline{b}\Delta)$ and similarly $c_A^1(\bar{e}) = \frac{1}{2}(\bar{b}\Delta)$. In this case since agent's effort level is observable and she is paid based on the

⁶We omit b as variable in the accuracy levels in notation henceforth.

effort level, the optimized level of agent's effort is actually derived from principal point of view. That is principal will choose effort level and also wage amount in a way which maximizes his total expected payoffs and also because of that principal lets effort level of agent equal zero when they develop B outline. The reason is that agent's effort is just cost in this situation. The following lemma summarizes the above discussion when principal invests b and finds that the future true state is B' .

Lemma 1 *In the contract in which principal hires the agent, upon finding future state as B' , he offers wage contract depending on his type and the agent follows the recommended effort levels by each type of principal, $e(\underline{b})$ and $e(\bar{b})$, respectively.*

The above lemma tells us in the case which principal hires the agent information superiority of principal does not lead to rent payoff. Now, having the optimal behavior in the stage-1, the optimization of principal's problem gives us $T'(b_1^*) = \rho\Delta r = \rho\Delta\frac{1}{2}[\alpha\bar{e}\bar{b} + (1-\alpha)\underline{e}\underline{b}]$. The higher level of accuracy will result in higher level of cognition investment. So this means that higher productivity of cognition technology (higher accuracy) will result in higher level of cognition investment or more complete contract.

4.2 Joint Production

In this part we analyze the situation in which principal not only develops the outline in collaboration with agent but also they jointly produce the final product. In this case similar to the basic model they will compromise on the sharing rule contract which actually has an equivalent wage contract like before. For any level of cognition investment, there are two levels for outline accuracy (in case of $b = 1$ these two levels are equal as it was explained above). Now the question is that does information superiority of principal have effect on the game's equilibrium? We first show that for a given cognition investment b , when P finds B' , it is optimal for principal to report either $\underline{b}(b)$ or $\bar{b}(b)$ not any other level of accuracy (say $\underline{b}(b')$ or $\bar{b}(b')$). And we also show that for low and high values of cognition investment, b , the agent's action is independent of principal's message and for middle values of b the agent chooses action based on the report.

Lemma 2 *For any given value of cognition investment b , principal does not have incentive to misreport $\underline{b}(b)$ and $\bar{b}(b)$.*

Proof Since for any level of cognition investment, b , we have two correspondent accuracy

levels ($\underline{b}(b)$ and $\bar{b}(b)$) and given that agent will choose effort level optimally⁷ the optimal low effort level is determined by

$$\max_{\{e\}} \{V - c_P - c(e) - (1 - \frac{eb}{2})a_1\}$$

and optimal high effort level is determined by

$$\max_{\{e\}} \{V - c_P - c(e) - (1 - \frac{e\bar{b}}{2})a_1\}$$

These optimizations give us $c'(\underline{e}) = \frac{ba_1}{2}$ and $c'(\bar{e}) = \frac{\bar{b}a_1}{2}$, respectively. Now suppose principal reports different accuracy level, say \bar{b}' , then agent will choose high effort level as $c'(\bar{e}') = \frac{\bar{b}'a_1}{2}$ which is different from optimal effort level and also results in cost amount which is different from optimal amount. So principal does not have any incentive to misreport $\bar{b}(b)$. Similarly he does not have incentive to misreport $\underline{b}(b)$ ⁸ ■

Now we turn to the characterization of the equilibrium in this case. The following proposition divides cognition investment levels in three intervals in which the equilibrium behaviors of players are different.

Proposition 3 *There is $\tilde{c}(b) = \frac{2}{a_1} \frac{c_A(\bar{e}) - c_A(\underline{e})}{\bar{e} - \underline{e}}$ such that;*

- i. If $\bar{b} \leq \tilde{c}(b)$ the agent chooses low effort level independent of messages from different types of principal.*
- ii. If $\underline{b} \geq \tilde{c}(b)$ the agent chooses high effort level independent of messages from different types of principal.*
- iii. For the middle values of b there exists a unique separating equilibrium in which high type sends \bar{b} and low type sends \underline{b} and agent chooses \bar{e} and \underline{e} , respectively.*

Proof Please see Appendix-B. ■

The following figure illustrates above proposition.

The reason of why this type of behavior happens is that agent will take into account the cost of taking high effort when the cognition investment is low. That is trade off between high cost and the benefit of higher probability of developing appropriate design

⁷Note that in this case agent will choose optimal effort level by herself. Of course optimization by principal would give same result because of sharing rule contract.

⁸What is essential in the proof is that optimal effort level is referred to the situation in which agent gets low effort level for low type principal and high effort level for high type principal.

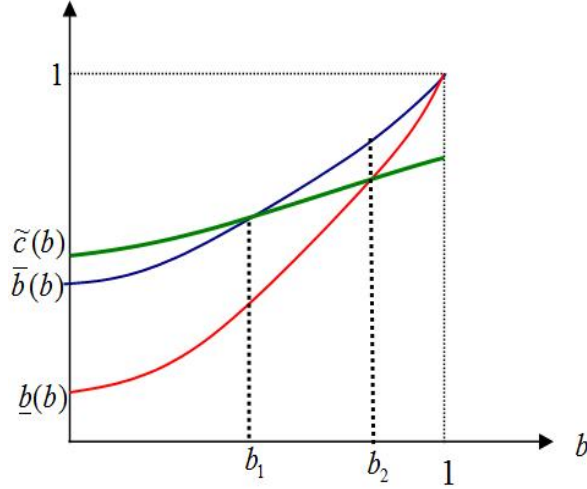


Figure 3: Different levels of b which result in different types of equilibrium.

will result in low effort level. The similar argument is true when the cognition investment is high. In that case the trade off between higher cost of effort level and higher probability of appropriate design will result in high effort level.

Another point which is worth to pay attention is that optimal effort level in hiring contract is higher in comparison to joint production contract. By looking at the optimal effort level in hiring contract, $c_A'(\underline{e}) = \frac{1}{2}(b\Delta)$ and $c_A'(\bar{e}) = \frac{1}{2}(\bar{b}\Delta)$, and comparing them with correspondent effort level in joint production contract, $c_A'(\underline{e}) = \frac{1}{2}(b a_1)$ and $c_A'(\bar{e}) = \frac{1}{2}(\bar{b} a_1)$, we find that, given b , in the former case the optimal effort level is higher. Since the effort level contributes in increasing the probability of getting intended design, in hiring contract the marginal effect of effort is getting Δ with higher probability while in the joint production contract its marginal effect is getting a_1 . In other words, in the hiring contract marginal benefit of higher effort is higher than joint production contract. The following lemma summarizes the above arguments.

Lemma 3 *For each cognition investment level, the optimal effort level of agent in hiring contract is higher respect to joint production contract.*

Since we have different types of equilibrium for different values of cognition investment, b , the payoff function is different in different levels of cognition investment. To analyze it in general sense we do not restrict ourselves to subintervals for each of these functions. That is we treat them as three (one for each interval) functions which are defined on the interval $b \in [0, 1]$. Similar to basic model we can proceed by introducing a payoff equivalent wage

contract here. Suppose principal finds that future state is B' and $b \in [0, b_1]$ then he would get

$$\alpha\left[\frac{e\bar{b}}{2}\sigma(V-c) + \left(1 - \frac{e\bar{b}}{2}\right)\sigma(V-c-a_1)\right] + (1-\alpha)\left[\frac{e\underline{b}}{2}\sigma(V-c) + \left(1 - \frac{e\underline{b}}{2}\right)\sigma(V-c-a_1)\right]$$

And if he offers wage contract $w(B')$ the payoff is

$$\begin{aligned} & \alpha\left[\frac{e\bar{b}}{2}(V-c_p-w(B')) + \left(1 - \frac{e\bar{b}}{2}\right)(V-\Delta-c_p-w(B') + \sigma(\Delta-a_1))\right] \\ & + (1-\alpha)\left[\frac{e\underline{b}}{2}(V-c_p-w(B')) + \left(1 - \frac{e\underline{b}}{2}\right)(V-\Delta-c_p-w(B') + \sigma(\Delta-a_1))\right] \end{aligned}$$

By equating these two values we find the wage contract (similar to basic model), when principal finds that the future state is B' , as $w(B') = \beta[V-c-(1-\frac{e}{2}(\alpha\bar{b}+(1-\alpha)\underline{b}))\Delta] + c_A$.

With the same argument as basic model the wage offer when principal does not find anything is $w(B) = \beta(V-c-\hat{\rho}\Delta) + c_A$.

Principal's payoffs when $b \in [0, b_1]$ is as follows;

$$\begin{aligned} U_1(b) = & -T(b) + \rho b[V-c_p-w(B')-\Delta + \sigma(\Delta-a_1)] + \frac{e}{2}(\Delta - \sigma(\Delta-a_1))(\alpha\bar{b} + (1-\alpha)\underline{b}) \\ & + \rho(1-b)[V-c_p-\Delta-w(B) + \sigma(\Delta-a_2)] + (1-\rho)[V-c_p-w(B)] \end{aligned}$$

With the same arguments we can derive the payoff function of principal when $b \in (b_1, b_2]$ and $b \in (b_2, 1]$. The only difference between these three payoff functions is related to $w(B')$. That is when $b \in [b_1, b_2]$ the wage offer is $w(B') = \beta[V-c-(1-\frac{1}{2}(\alpha\bar{e}b+(1-\alpha)\underline{e}b))\Delta] + \alpha\bar{c}_A + (1-\alpha)\underline{c}_A$ And similarly when $b \in [b_2, 1]$ we have $w(B') = \beta[V-c-(1-\frac{e}{2}(\alpha\bar{b}+(1-\alpha)\underline{b}))\Delta] + c_A$. So $U_2(b)$ and $U_3(b)$ can be driven in similar way of U_1 but of course the substituted $w(B')$ is different in three cases as mentioned above.

The following figure shows the three different payoff functions.⁹This means that the optimal value of any interval occurs in that interval(note that the main reason of this

⁹One can show that the relation between values of these three function is the same as shown in each interval for all possible payoff functions.

To show $d = (U_1(b) - U_2(b)) \geq 0$, take the difference of these two payoff functions when $b \in [0, b_1]$. Then

$$\begin{aligned} d &= \alpha(c(\bar{e}) - c(\underline{e}) - (\bar{e} - \underline{e})\frac{\sigma\bar{b}a_1}{2}) \\ \Rightarrow \frac{d}{\alpha(\bar{e} - \underline{e})} &= \frac{c(\bar{e}) - c(\underline{e})}{\bar{e} - \underline{e}} - \frac{\sigma\bar{b}a_1}{2} \end{aligned}$$

Given that $\bar{b} \leq \tilde{c}(b)$ we have

$$\frac{d}{\alpha(\bar{e} - \underline{e})} \geq \beta\left(\frac{c(\bar{e}) - c(\underline{e})}{\bar{e} - \underline{e}}\right) \geq 0$$

And in similar way it can be shown that when $b \in [0, b_1], U_2 > U_1$; $b \in [b_1, b_2], U_2 \geq (U_1, U_2)$ and when $b \in (b_2, 1), U_3 > U_2 > U_3$. And also the the relation between the slopes at b_1 and b_2 can be shown that $U'_1(b_1) < U'_2(b_1)$ and $U'_2(b_2) < U'_3(b_2)$ by the analysis which comes in the next page.

conclusion comes from proposition 4).

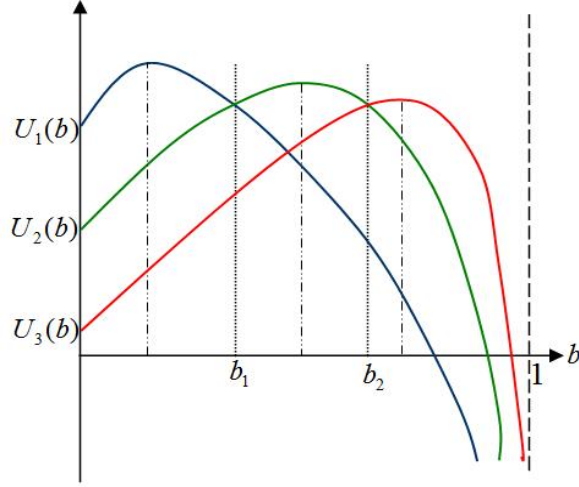


Figure 4: U_i is the payoffs of principal based on the equilibrium in i 'th interval.

Lemma 4 *In both types of contract the optimal effort level when principal and agent develop the design based on outline B is zero and so is the optimal cost.*

Proof Since developing the B outline does not need explicitly the effort level of agent, to minimize the cost it is optimal to choose it at zero level in both type of contracts. ■

The intuition of this lemma is that the principal knows how to develop based on the outline B and he actually does not need agent's help for that. However, if principal wants to develop based on the outline B' then he needs agent. You can think of outline B' as a path breaking technology in the industry.

By substituting of $w(B')$ and $w(B)$ in each relevant U_i th and taking derivative respect to b we get three equations which indicate optimal condition for each intervals. The followings are those equations under assumption 1.

If $b \in [0, b_1]$

$$U'_1(b) = -T'(b) + \rho\sigma(a_2 - (1 - \frac{e}{2}[\alpha\bar{b} + (1 - \alpha)\underline{b}])a_1) + \rho\beta\Delta(1 - \hat{\rho}) - \sigma\rho c(\underline{e})$$

if $b \in (b_1, b_2]$

$$U'_2(b) = -T'(b) + \rho\sigma(a_2 - (1 - \frac{1}{2}[\alpha\bar{e}\bar{b} + (1 - \alpha)\underline{e}\underline{b}])a_1) + \rho\beta\Delta(1 - \hat{\rho}) - [\alpha c(\bar{e}) + (1 - \alpha)c(\underline{e})]$$

and if $b \in (b_2, 1]$

$$U'_3(b) = -T'(b) + \rho\sigma(a_2 - (1 - \frac{\bar{e}}{2}[\alpha\bar{b} + (1 - \alpha)\underline{b}])a_1) + \rho\beta\Delta(1 - \hat{\rho}) - \sigma\rho c(\bar{e})$$

So the optimal cognition investment satisfies $U'_i(b_i^*) = 0$ for each of the specified interval.

Now the question is that given the optimal cognition investment (i.e. b_i^*) which level should be chosen by principal. This is a straightforward analysis. That is we just need to compare the payoff values of principal in the three intervals. Before going to the proposition which characterizes that we introduce a new notation which virtually captures the *Gain* to the principal which is actually negative of loss incurred in addition to cognition cost. Define $G_i(b_i^*)$ as;

$$G_i(b_i^*) = \beta\Delta[\rho b_i^* - (1 - \rho b_i^*)\hat{\rho}] - \sigma\rho[b_i^* a_1(1 - r_i) + a_2(1 - b_i^*) + b_i^* c_i^*]$$

where $r_1 = \frac{e}{2}(\alpha\bar{b} + (1 - \alpha)\underline{b})$, $r_2 = \frac{1}{2}(\alpha\bar{e}\underline{b} + (1 - \alpha)\underline{e}\underline{b})$, and $r_3 = \frac{\bar{e}}{2}(\alpha\bar{b} + (1 - \alpha)\underline{b})$ when $b \in [0, b_1]$, $b \in (b_1, b_2]$, and $b \in (b_2, 1]$, respectively. We also have $c_1^* = c(\underline{e})$, $c_2^* = \alpha c(\bar{e}) + (1 - \alpha)c(\underline{e})$, and $c_3^* = c(\bar{e})$.

This gain function contains two parts. First part is the (adjusted)part of Δ which goes to the agent and the second part is just the (negative) portion of cost that principal should pay in cases which they need to adjust. The following proposition characterizes the optimal behavior of principal.

Proposition 4 *Given the optimal values of cognition investment in each interval (i.e. b_i^*) principal will choose;*

- i) b_1^* ; if $\frac{T_2 - T_1}{G_2 - G_1} > 1$ and $\frac{T_3 - T_1}{G_3 - G_1} > 1$,
- ii) b_2^* ; if $\frac{T_1 - T_2}{G_1 - G_2} > 1$ and $\frac{T_3 - T_2}{G_3 - G_2} > 1$,
- iii) b_3^* ; if $\frac{T_1 - T_3}{G_1 - G_3} > 1$ and $\frac{T_2 - T_3}{G_2 - G_3} > 1$.

Proof Please see Appendix-B. ■

This proposition says that principal chooses b_1^* if choosing b_2^* and b_3^* are more costly. The ratio in the proposition captures the relative changes in cost to gain. For example $\frac{T_2 - T_1}{G_2 - G_1} > 1$ means that cost of changing from b_1^* to b_2^* is more than gains of it.

We are more interested to look at what are the effects of accuracy levels. Here we consider two conceptually different situations. The first situation in which we call it more accurate outline happens if for any given level of cognition investment the low and high level of accuracies are higher. However, in the second situation we look for more volatile accuracy levels. We call accuracy levels more volatile respect to a given accuracy levels

if for any given cognition investment the high accuracy is higher and the low accuracy is lower.

The first observation is that the results of basic model are replicable here. To see that, we explain what is the implicit assumption in the basic model. In the basic model the assumption is that $\forall b \in [0, 1]$, we have

$$\bar{b}(b) = \underline{b}(b) = 1$$

this will give us constant probability for developing appropriate design (effort level is constant in this situation). This means that our model can replicate most of the result in the literature in this special case. In general, more accurate outline will result in higher probability of developing appropriate design via higher accuracy and higher effort level. The second observation is that more volatile accuracy, by assuming quadratic form¹⁰ of agent's cost function and mean preserving¹¹, does not change equilibrium behaviors in the hiring contract while in the joint production contract it does have effect.

Since the cost function is quadratic form, mean preserving more volatile accuracy levels do not change optimal behavior of agent and essentially nothing will change. Even though more volatile accuracy levels do not change the optimal level of effort levels under the above assumptions but they will change the threshold of optimal behaviors. For example more volatile accuracy increases the threshold which agent chooses high effort level and decreases the threshold for low effort level as it is shown in the below figure.

Figure 4 suggests that similarity of equilibrium behavior of agents in hiring and joint production contracts increases with more volatile accuracy levels. That is by increasing volatility which results in higher b_2 and lower b_1 , the equilibrium behavior of agent in joint production looks more similar to equilibrium behavior in hiring contract. This analysis tells us that higher volatility in the accuracy reduces incentive to invest more to get advantage of high effort level in joint production contract.

Now we compare the completeness of these two contracts like what we did in the basic model. Here $r = \frac{1}{2}(\alpha\bar{b} + (1 - \alpha)\underline{b})$ is the probability of developing appropriate design at hiring and r_i , $i \in \{1, 2, 3\}$, is the similar probability at joint production.

¹⁰We impose this assumption in this part to get simpler analysis, and also this type of cost function has been used in the literature.

¹¹Suppose we have $\underline{b}^0(b)$, $\bar{b}^0(b)$ and $\underline{b}^1(b)$, $\bar{b}^1(b)$ in which $\underline{b}^1(b) < \underline{b}^0(b)$ and $\bar{b}^0(b) < \bar{b}^1(b)$ we call the more volatile accuracy levels mean preserving if

$$\alpha\bar{b}^0(b) + (1 - \alpha)\underline{b}^0(b) = \alpha\bar{b}^1(b) + (1 - \alpha)\underline{b}^1(b).$$

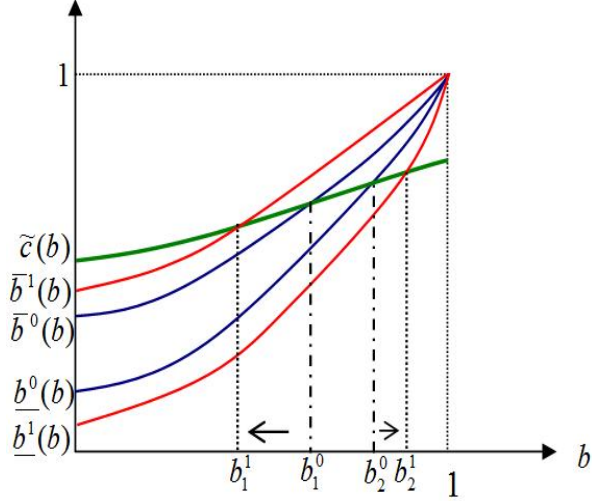


Figure 5: More volatile accuracy levels with mean preserving condition.

Proposition 5 *For high level complexity $r < \beta(1 - \rho)$ the hiring of agent is always more complete than joint production and for low level of complexity $r > \beta$ hiring the agent is more complete only if $\Delta > \frac{\sigma(a_2 - (1-r_i)a_1 - c_i)}{r - \beta}$.*

Proof The proof is similar to the proposition-1 and similar argument works for the cases when $\beta(1 - \rho) \leq r \leq \beta$. ■

We see similar behaviors in the cases with high complexity. However, in the case with low complexity the equilibrium behavior of agent in joint production will significantly change the likelihood of completeness of joint production design. The reason is that increasing the probability of appropriate design will increase the speciality gains which have positive effect on the completeness of joint production contract. Given the above analysis about the effect of changing in the accuracy levels, the higher(lower) is the accuracy levels(volatility) the higher is the likelihood of higher completeness of joint production contract. Since principal knows if he invests more than a threshold (i.e. $b > b_2$) the optimal behavior of agent will result in higher speciality gain.

4.3 Contractual Design

Now we proceed similar to what we did in basic model. We derive net payoffs as

$$\begin{aligned} \aleph(c_1, c_2, B, B', a_1, a_2, \Delta) = & \{-T(b_2^i) + (V - c_P) - \sigma\rho\Delta\gamma^i - \beta(V - c) - c_A^i + \rho\Delta\gamma^i + \sigma\rho\lambda^i + \rho\Delta r b_2^i\} \\ & - \{-T(b_1) + (V - c_P) - c_A + \rho\Delta r b_1\} \end{aligned}$$

similar to what we have seen in basic model, where $\gamma^i = (b_2^i(1 - r^i) + (1 - b_2^i)\hat{\rho})$ as the probability which the design is not appropriate and needs to be renegotiated in joint production, $\lambda^i = (b_2^i(1 - r^i)(\Delta - a_1) + (1 - b_2^i)(\Delta - a_2))$ as expected surplus in case of negotiation on inappropriate design, and $i \in \{1, 2, 3\}$ represents different equilibrium values of cognition investment in joint production. With similar argument principal chooses contracts based on the following proposition.

Proposition 6 *Principal will choose joint production if $\aleph > 0$, and hiring if $\aleph < 0$ and he is indifferent between two cases if $\aleph = 0$.*

So principal will choose joint production if

$$\{\rho\sigma\lambda^i + \rho\Delta r^i(b_2^* - b_1^*) + \beta\rho\Delta\gamma^i\} + \{(-T(b_2) + T(b_1)) - \beta(V - c) - (c_A^i - c_A)\} > 0$$

4.4 Discussion

One of the interesting analysis to understand the effects of accuracies and asymmetric information is equilibrium behaviors of players in the joint production contract. Suppose the accuracy level was common knowledge, then the payoffs of principal would be similar to the $U2(b)$. This means that asymmetric information on the accuracy levels will result in higher (b_3^*) or lower (b_1^*) depending on the conditions specified in the proposition 4.

5 Conclusion

We analyzed the contract design problem in production. We considered two types of contracts. Our analysis suggests that cognition investment level depends on the contract's form. In particular, when the developing stage of the production is complex (low probability of developing appropriate design), the joint production contract is more complete and if the complexity is low then the higher speciality gain, the higher incomplete is the joint production contract. We found that when developing stage is complex marginal decrease in the complexity increases tendency to choose hiring contract.

Breaking the dichotomy of effect of cognition investment on before and after contract behaviors, we investigate effect of cognition investment on the accuracy of outline and its effect on the equilibrium behavior of principal and agent. While accuracy level does not affect on the equilibrium behavior of agent in hiring contract, it does have effect on the agents behavior in joint production. For low and high levels of cognition investment

agent's effort level is independent of principal report of accuracy while for middle level of cognition investment principal report of accuracy level affects agents behavior.

The more accurate outline will result in higher level of cognition investment and vice versa. However, the more volatile accuracy (under some condition) will reduce principal's incentive for higher cognition investment. Higher volatility in the accuracy makes optimal behavior of agent in joint production more similar to hiring contract. That is it destroys collaborative behavior of agent for high cognition investment level, other condition fixed.

6 Appendix

6.1 Appendix-A

$w(B')$;

By equating what P would get having found design outline B' and what he actually receives after offering the $w(B')$;

$$\sigma(V - c - (1 - r)a_1) = V - c_P - w(B') - (1 - r)(\Delta - \sigma(\Delta - a_1)) \Rightarrow$$

$$w(B') = (1 - \sigma)V - c_P - (1 - \sigma)(1 - r)\Delta + \sigma c_A$$

This will give us the $w(B')$.

$w(B)$;

Same argument as above we have

$$\sigma(V - c - \hat{\rho}a_2) = V - c_P - w(B) - \hat{\rho}(\Delta - \sigma(\Delta - a_2)) \Rightarrow$$

$$w(B) = (1 - \sigma)V - c_P - (1 - \sigma)\hat{\rho}\Delta + \sigma c_A$$

This also gives us the value of $w(B)$. To find optimal value of b_2 by taking derivative respect to b_2 we have

$$-T'(b) + \rho r(V - c - \beta(V - c - (1 - r)\Delta)) + \rho(1 - r)(V - \Delta - c - \beta(V - c - (1 - r)\Delta) + \sigma(\Delta - a_1))$$

$$- \rho(V - \Delta - c - \beta(V - c - \hat{\rho}\Delta + \sigma(\Delta - a_2))) = 0 \quad \Rightarrow$$

$$-T'(b) + \rho\beta(1 - r)\Delta + \rho\Delta - \rho\beta\hat{\rho}\delta - \rho\sigma(\Delta - a_2) - \rho(1 - r)\Delta + \rho(1 - r)\sigma(\Delta - a_1) = 0 \quad \Rightarrow$$

$$T'(b_2) = \rho a_2 + \rho\beta\Delta - \rho\beta a_2 - \rho\beta\hat{\rho}\Delta - \rho(1 - r)a_1 + \rho(1 - r)\beta a_1$$

This will give us the optimal value of b_2 .

Hiring or Joint Production?;

We can write the optimal net payoffs from proposing joint production contract instead of hiring as follows;

$$\begin{aligned} \aleph(c_1, c_2, B, B', a_1, a_2, \Delta) = & \{-T(b_2) + \rho b_2 r(V - c_P - \beta(V - c - (1 - r)\Delta - c_A)) + \rho b_2 (1 - r)(V - \Delta \\ & - c_P - \beta(V - c - (1 - r)\Delta) - c_A + \sigma(\Delta - a_1)) + \rho(1 - b_2)(V - \Delta - c_P - \beta(V - c - \hat{\rho}\Delta) - c_A + \sigma(\Delta - a_2)) \\ & + (1 - \rho)(V - c_P - \beta(V - c - \hat{\rho}\Delta))\} - \{-T(b_1) + \rho b_1 r(V - c_P) + \rho b_1 (1 - r)(V - \Delta - c_P) + \rho(1 - b_1)(V \\ & - c_P - \Delta) + (1 - \rho)(V - c_P) - c_A\} \end{aligned}$$

A bit manipulating the algebra we get the simplified terms for payoffs in each contract. The payoffs to principal when he offers hiring;

$$-T(b_1) + V - c_P - \rho\Delta(b_1(1-r) + (1-b_1)) - c_A$$

and the payoffs when he offers joint production

$$\begin{aligned} & -T(b_2) + V - c_P - \beta(V - c) - c_A + \rho\beta\Delta(b_2(1-r) + (1-b_2)\hat{\rho}) + \rho\sigma(b_2(1-r)(\Delta - a_1) + (1-b_2)(\Delta - a_2)) - \\ & \rho\Delta(b_2(1-r) + (1-b_2)) \end{aligned}$$

The $\aleph(\cdot)$ is the difference of these two values.

Proposition-1 Cont.

When $\beta(1-\rho) < r < \beta$ the optimal value of cognition investment in joint production (b_2^*) would matter. In particular there are two values of b , ($b^0 < b^1$) such that;

$\Delta = \frac{\sigma(a_2 - (1-r)a_1)}{r - \beta(1 - \hat{\rho}(b^0))}$ and $r = \beta(1 - \hat{\rho}(b^1))$. So if $b < b^1$ then $\frac{\sigma(a_2 - (1-r)a_1)}{r - \beta(1 - \hat{\rho}(b))} > 0$ and vice versa.

By the similar argument of proposition-1 in the text we have;

If $b_2^ < b^0$ and $b_2^* > b^1$ then $b_1^* > b_2^*$ and for the other values of b_2^* we have $b_1^* < b_2^*$.*

6.2 Appendix-B

Payoffs;

Payoffs of principal and agent is based on the sharing rule which σ portion of them go to principal and $(1-\sigma)$ portion of them go to agent. In the following we omit these portion factor and just write total payoffs in each final node of the game tree. The payoff function $\pi(e, b|\hat{b})$ indicates total payoff in which agent chooses action e and principal type is b condition on agent receives \hat{b} . Having this the payoffs are as follow;¹²

$$1-\pi(\bar{e}, \bar{b}|\bar{b}) = \pi(\bar{e}, \bar{b}|\underline{b}) = V - c_P - c(\bar{e}) - (1 - \frac{\bar{e}\bar{b}}{2})a_1$$

$$2-\pi(\underline{e}, \bar{b}|\bar{b}) = \pi(\underline{e}, \bar{b}|\underline{b}) = V - c_P - c(\underline{e}) - (1 - \frac{\underline{e}\bar{b}}{2})a_1$$

$$3-\pi(\bar{e}, \underline{b}|\bar{b}) = \pi(\bar{e}, \underline{b}|\underline{b}) = V - c_P - c(\bar{e}) - (1 - \frac{\bar{e}\underline{b}}{2})a_1$$

$$4-\pi(\underline{e}, \underline{b}|\bar{b}) = \pi(\underline{e}, \underline{b}|\underline{b}) = V - c_P - c(\underline{e}) - (1 - \frac{\underline{e}\underline{b}}{2})a_1$$

Condition on α ;

In this part we derive the probability in which agent is indifferent between \underline{e} and \bar{e} . Agent's payoffs when she chooses \bar{e} (we remove $(1-\sigma)$ since it makes no difference here)

$$j(-c(\bar{e}) - a_1 + \frac{\bar{e}\bar{b}}{2}a_1) + (1-j)(-c(\bar{e}) - a_1 + \frac{\bar{e}\underline{b}}{2}a_1)$$

¹²We remove subscript from agent cost hoping makes no confusion.

And when she chooses \underline{e}

$$j(-c(\underline{e}) - a_1 + \frac{e\bar{b}}{2}a_1) + (1-j)(-c(\underline{e}) - a_1 + \frac{eb}{2}a_1)$$

equating these two payoffs we have

$$j[(\bar{e} - \underline{e})(\bar{b} - \underline{b})a_1] = 2(c(\bar{e}) - c(\underline{e})) - \underline{b}(\bar{e} - \underline{e})a_1$$

$$\Rightarrow j = \frac{2}{a_1} \frac{c(\bar{e}) - c(\underline{e})}{\bar{e} - \underline{e}} \frac{1}{\bar{b} - \underline{b}} - \frac{\underline{b}}{\bar{b} - \underline{b}}$$

So if $\alpha > j$ agent will choose \bar{e} and if $\alpha < j$ he will choose \underline{e} in case of $\alpha = j$ she is indifferent between these two actions.

Proof of Proposition 4;

Let $\tilde{c}(b)$ be $\tilde{c}(b) = \frac{2}{a_1} \frac{c(\bar{e}) - c(\underline{e})}{\bar{e} - \underline{e}}$ we will go through of all cases as result of different values of \bar{b} and \underline{b} .

C-1; $\bar{b} < \tilde{c}$;

First note that \underline{b} is also $\underline{b} < \tilde{c}$. In this case agent will choose \underline{e} independent of principal's report. This is true since given that agent chooses \underline{e} , high type principal's payoff is the same for any messages he sends ($\pi(\underline{e}, \bar{b}|\bar{b}) = \pi(\underline{e}, \bar{b}|\underline{b})$). And similarly for low type principal's payoffs ($\pi(\underline{e}, \underline{b}|\bar{b}) = \pi(\underline{e}, \underline{b}|\underline{b})$). This argument also shows that we can not have separating equilibrium. To see whether we have pooling equilibrium¹³ suppose both types send \bar{b} . If $\alpha > j$ agent chooses \bar{e} . In off-equilibrium path if $pr(\bar{b}|\underline{b}) = \mu > j$, that is belief of agent in the off-equilibrium path in which principal is high type given message is \underline{b} , then agent will choose \bar{e} this is not equilibrium since high type principal can mix over messages. If $\mu < j$ then agent chooses \underline{e} which is not equilibrium since high type principal chooses \underline{b} . When $\mu = j$ pooling equilibria is not sustainable. If $\alpha < j$ then agent chooses \underline{e} and in off-equilibrium path when $\mu > j$ agent chooses \bar{e} . If $\alpha = j$ agent will randomize between \bar{e} and \underline{e} and belief in the off-equilibrium path is $\mu > j$ which agent chooses \bar{e} . So we have pooling equilibrium if $\alpha < j$.

C-2; $\bar{b} \geq \tilde{c}$ and $\underline{b} < \tilde{c}$

Separating Equilibria;

By separating equilibria we look for the situation in which high type principal sends \bar{b} and low type principal sends \underline{b} . Under this case agent chooses high effort level when she receives \bar{b} and chooses low effort level when she receives \underline{b} . Under this condition no type

¹³We are essentially interested in the payoffs in the resulting equilibria. Since in mixing equilibria(if there is any) the payoff is the same as either separating or pooling equilibria we do not analyze them here.

has incentive for profitable deviation. The reason is that if high type principal sends \bar{b} the agent payoffs by taking high effort is $-c(\bar{e}) - a_1 + \frac{\bar{e}b}{2}a_1$ and her payoffs by taking low effort is $-c(\underline{e}) - a_1 + \frac{\bar{e}\underline{b}}{2}a_1$. Having the above condition over \bar{b} agent will choose high effort level and similarly if low type principal sends \underline{b} then by choosing high effort agent gets $-c(\bar{e}) - a_1 + \frac{\underline{e}b}{2}a_1$ and low effort she gets $-c(\underline{e}) - a_1 + \frac{\underline{e}\underline{b}}{2}a_1$. Under specified condition agent will choose \underline{e} . This constructs our separating equilibrium.

Pooling Equilibria;

Suppose both types send \bar{b} , if $\alpha > j$ then agent will choose \bar{e} . In the off-equilibrium path; if $\mu > j$ then agent will choose \bar{e} in this situation principal is indifferent between sending two messages. If $\mu < j$ then agent chooses \underline{e} in this case low type principal deviates to send \underline{b} and if $\mu = j$ the pooling equilibria is not sustainable. If $\alpha < j$ then agent chooses \underline{e} . In the off-equilibrium path if $\mu > j$ then agent will choose \bar{e} . In this case \bar{b} deviates to \underline{b} . If $\mu < j$ then agent chooses \underline{e} . In this case high type principal is indifferent between sending two messages. If $\mu = j$ the pooling equilibria is not sustainable. And if $\alpha = j$ then agent is indifferent between \underline{e} and \bar{e} . In the off-equilibrium path for any belief $\mu \neq j$ it is optimal for one of the senders to deviate. For $\mu = j$ pooling equilibria is not sustainable. If both senders send \underline{b} the analysis is similar.

C-3; $\bar{b} > \tilde{c}$ and $\underline{b} \geq \tilde{c}$

Separating Equilibrium; Note that in this case $\alpha \geq j$. If high type sends \bar{b} and low type sends \underline{b} then agent chooses \bar{e} for both types of principal. So there is no separating equilibrium.

Pooling Equilibrium;

If both types send \bar{b} agent will choose \bar{e} . In off-equilibrium path for any $\mu > 0$ it is optimal for agent to choose \bar{e} (for $\mu = 0$ we have mix equilibria which we ignore here since it does not have effect on the equilibrium payoffs). ■

Proof of Proposition 5;

To prove this proposition we start when $b \in [0, b_1]$, (for other interval the procedure is similar). We have

$$\begin{aligned}
U_1(b_1^*) &= -T(b_1^*) + \rho b_1^* \{V - c_P - w(B') - \Delta + \sigma(\Delta - a_1) + \frac{\rho}{2} [(\Delta - \sigma(\Delta - a_1))(\alpha \bar{b} + (1 - \alpha)\underline{b})]\} \\
&\quad + \rho(1 - b_1^*) \{V - \Delta - c_P - w(B) + \sigma(\Delta - a_2)\} + (1 - \rho) \{V - c_P - w(B)\} \\
\Rightarrow U_1(b_1^*) &= -T(b_1^*) + V - c_P - \rho\Delta + \sigma\rho\Delta - \rho b_1^* w(B') - \sigma\rho b_1^* a_1 + \rho b_1^* r_1 (\Delta - \sigma(\Delta - a_1)) \\
&\quad - \rho(1 - b_1^*) w(B) - \sigma\rho(1 - b_1^*) a_2 - (1 - \rho) w(B)
\end{aligned}$$

Substituting $w(B)$ and $w(B')$ and by some simplification in algebra we get

$$\begin{aligned}
U_1(b_1^*) &= -T(b_1^*) + \sigma(V - c) - \rho\Delta + \sigma\rho\Delta + \beta\Delta(\rho b_1^*(1 - r_1) + (1 - \rho b_1^*)\hat{\rho}) - \sigma\rho(b_1^*a_1 + (1 - b_1^*)a_2) \\
&\quad + \rho b_1^* r_1(\Delta - \sigma(\Delta - a_1)) - \sigma\rho b_1^* c_1^* \\
&= -T(b_1^*) + \sigma(V - c) - \rho\Delta + \sigma\rho\Delta + [\rho b_1^* \Delta \beta(1 - \hat{\rho}) + \beta \Delta \hat{\rho}] - [\sigma\rho b_1^* a_1(1 - r_1) + \sigma\rho a_2(1 - b_1^*) + \sigma\rho b_1^* c_1^*] \\
&= -T(b_1^*) + \sigma(V - c) - \rho\Delta + \sigma\rho\Delta + \beta\Delta[\rho b_1^* - (1 - \rho b_1^*)\hat{\rho}] - [\sigma\rho b_1^* a_1(1 - r_1) + \sigma\rho a_2(1 - b_1^*) + \sigma\rho b_1^* c_1^*] \\
&= -T(b_1^*) + \sigma(V - c) - \rho\Delta + \sigma\rho\Delta + G_1(b_1^*)
\end{aligned}$$

Now to see that when $U_1(b_1^*) > U_2(b_2^*)$ and $U_1(b_1^*) > U_3(b_3^*)$ we need to have $\frac{T_2 - T_1}{G_2 - G_1} > 1$ and $\frac{T_3 - T_1}{G_3 - G_1} > 1$. This proves part *i*) of proposition. The other two parts can be proven by the same approach. ■

6.3 Appendix-C

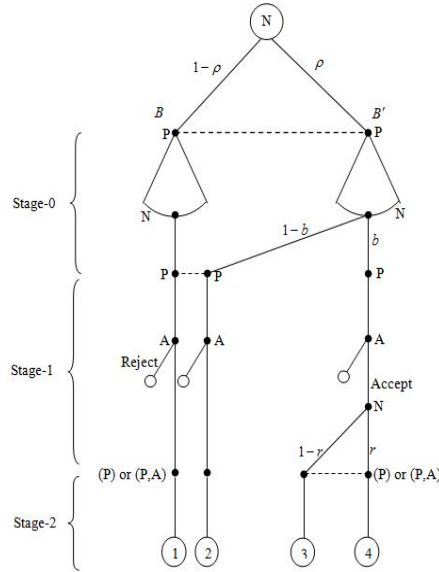


Figure 6: Extensive form game of Basic Model.

The numbers at the final nodes indicate the payoffs. The payoffs depend on the contract (i.e. *hiring* or *joint production*). If the contract is *hiring*;

$$1 \equiv ((1 - \rho)(V - c_P), (1 - \rho)c_A^1);$$

$$2 \equiv (\rho(1 - b)(V - c_P - \Delta), \rho(1 - b)c_A^1);$$

$$3 \equiv (\rho b(1-r)(V - c_P - \Delta), \rho b(1-r)c_A^1);$$

$$4 \equiv (\rho br(V - c_P), \rho br c_A^1).$$

If the contract is *joint production*;

Given $w(B') = \beta(V - c - (1-r)\Delta) + c_A$ and $w(B) = \beta(V - c - \hat{\rho}\Delta) + c_A$ then;

$$1 \equiv ((1-\rho)(V - c_P - w(B)), w(B));$$

$$2 \equiv (\rho(1-b)(V - \Delta - c_P - w(B) + \sigma(\Delta - a_2)), w(B));$$

$$3 \equiv (\rho b(1-r)(V - \Delta - c_P - w(B') + \sigma(\Delta - a_1)), w(B'));$$

$$4 \equiv (\rho br(V - c_P - w(B')), w(B')).$$

7 References

- 1-**Bajari, Patrick; Tadelis, Steven** , 2001;"Incentive versus Transaction Costs: a Theory of Procurement Contracts", Rand Journal of Economics.
- 2-**Cremer, Jacques; Khalid,Fahad; Rochet, Jean-Charls**,1998a;"Contract and Productive Information Gathering", Games and Economic Behavior.
- 3-**Cremer, Jacques; Khalid,Fahad; Rochet, Jean-Charls**,1998b;"Strategic Information Gathering before a contract is offered", Journal of Economic Theory.
- 4-**Hart, Oliver; Moore, John**,1988;"Incomplete Contracts and Renegotiation", Econometrica.
- 5-**Hart, Oliver; Moore, John**,1999;"Foundations of Incomplete Contracts", Review of Economic Studies.
- 6-**Herold, Florian**,2010;"Contractual incompleteness as a signal of trust", Games and Economic Behavior.
- 7-**Kessler, Anke**,1998;"The Value of Ignorance", The RAND Journal of Economics.
- 8-**Klein, Peter**,2004;"The Make-or-Buy Decision; Lessons from Empirical Studies", Working Paper.
- 9-**Krahmer, Daniel; Strausz, Roland**,2009;"Optimal Procurement Contracts with PreProject", Working Paper.
- 10-**Kunimoto, Takashi**,2008;"Indescribability and asymmetric information at the contracting stage", Economics Letter.
- 11-**Lafontaine, Francine; Slade, Margaret**, 2007; "Vertical Integration and Firm Boundaries: The Evidence", Journal of Economics Literature.
- 12-**Lewis, Tracy R.; Sappington, David E.**,1997;"Information Management in Incentive Problems", Journal of Political Economy.
- 13-**Maskin, Eric; Tirole, Jean**,1999;"Unforeseen Contingencies and Incomplete Contracts", Review of Economic Studies.
- 14-**Robertson, Thomas; Gattignon, Hubert**,1998;"Technology Development Mode: A Transaction Cost Conceptualization", Strategic Management Journal.
- 15-**Schmitz, Patrick**, 2008; "Information gathering and the hold-up problem in a complete contracting framework", Economics Letter.
- 16-**Segal, Ilya** ,1999;"Complexity and Renegotiation: A Foundation for Incomplete Contracts", Review of Economic Studies.
- 17-**Spier, Kathryn**,1992;"Incomplete Contracts and Signalling", The RAND Journal of Economics.

18-**Tirole, Jean**,1999;"Incomplete Contracts; where do we stand?" , Econometrica.

19-**Tirole, Jean**, 2009;"Cognition and Incomplete Contracts", American Economic Review.

20-**Ryall, Michael D.; Sampson, Rachelle C.**, 2003; "Do Prior Alliances Influence Contract Structure? Evidence from Technology Alliance Contracts", Working Paper.

21-**Zhao, Xiaojian**,2009;"Strategic Mis-Selling and Pre-Contractual Cognition", Working Paper.